Parametric Amplification of Density Perturbation in the Oscillating inflation

A.Taruya *

Department of Fundamental Sciences, FIHS, Kyoto University, Kyoto, 606-8501, Japan

Abstract

We study the adiabatic density perturbation in the *oscillating inflation*, proposed by Damour and Mukhanov. The recent study of the cosmological perturbation during reheating shows that the adiabatic fluctuation behaves like as the perfect fluid and no significant amplification occurs on super-horizon scales. In the oscillating inflation, however, the accelerated expansion takes place during the oscillating stage and there might be a possibility that the parametric amplification on small scales affects the adiabatic long-wavelength perturbation. We analytically show that the density perturbation neglecting the metric perturbation can be amplified by the parametric resonance and the instability band becomes very broad during the oscillating inflation. We examined this issue by solving the evolution equation for perturbation numerically. We found that the parametric resonance is strongly suppressed for the long wave modes comparable to the Hubble horizon. The result indicates that the metric perturbation plays a crucial role for the evolution of scalar field perturbation. Therefore, in the single field case, there would be no significant imprint of the oscillating inflation on the primordial spectrum of the adiabatic perturbation. However, it could be expected that the oscillating inflation in the multi-field system gives the enormous amplification on large scales, which may lead to the production of the primordial black holes.

PACS:98.80.Cq

I. INTRODUCTION

Dynamics of the coherently oscillating scalar field play an important role in the early stage of the universe and has been studied by many authors. Specifically, the significance of parametric resonance is found in the recent investigation of the reheating process [1]. Due to the coherent oscillation of the background inflaton field, the light boson field or the fluctuation of the inflaton itself is amplified through the non-linear interaction or the self-interaction [2,3]. The efficiency of the parametric resonance could have the important implication for GUT scale baryogenesis [4].

From the viewpoint of the large scale structure in the early universe, several authors investigated the influence of parametric resonance on the primordial density perturbation during the coherently oscillating stage after inflation. In the framework of cosmological perturbation, the single field model is studied in Ref. [5,6]. The analysis is also extended to the two-field model by Taruya and Nambu [7]. These analyses are mainly focused on the long wavelength perturbation, which significantly differs from the one neglecting the metric perturbation. Nambu and Taruya [6] investigate the Mukhanov's gauge-invariant variable and found that the evolution equation for perturbation can be reduced to the Mathieu type equation. Although the Mathieu equation itself has the exponential instability, the instability of the perturbation is very different from this and the final result is fully consistent with Kodama and Hamazaki [5]. The important conclusion of these analyses is that the density perturbation on super-horizon scales behaves like as the perfect fluid unless the iso-curvature perturbation becomes dominant. Therefore, as far as the adiabatic perturbation is concerned, the primordial power spectrum on large scales does not suffer from the significant amplification by the parametric resonance. However, there remains a possibility that the parametric resonance can appear well inside the Hubble horizon. For example, consider the massless self-interacting inflaton. Neglecting the metric perturbation, it is known that the fluctuation of the inflaton field has the instability mode whose wavelength is much smaller than the Hubble horizon [1,3].

The aim of the present paper is to clarify the perturbaton on sub-horizon scales taking into account the gravitational perturbation and understand the cosmological implication of the parametric amplification to the large scale structure. As usual, the universe expands decelerately during the coherently oscillating stage and the parametric resonance inside the Hubble horizon cannot affect the density perturbation on super-horizon scales. However, Damour and Mukhanov recently considered the model with the non-convex type potential [9]. In this model, taking the time average, the

*e-mail: ataruya@vishnu.phys.h.kyoto-u.ac.jp

potential energy of the inflaton becomes large compared to the kinetic energy and the accelerated expansion can take place in the oscillating stage. Liddle and Mazumdar called it *oscillating inflation* and confirmed this fact numerically [10]. As was suggested by Damour and Mukhanov, it could be expected that the very broad parametric resonance occurs and the quantum fluctuation is enormously amplified. This implies that the adiabatic metric perturbation with the large amplitude might be produced on super-horizon scales in the course of the accelerated expansion, which can lead to the different conclusion from the previous results [5,6]. Hence, there exists a possibility that the cosmological objects such as the primordial black holes may be formed by the large amplitude of the metric perturbation.

In this paper, to explore this possibility, we study the cosmological perturbation in the oscillating inflation and investigate the efficiency of the parametric resonance. We describe the model in Sec.II. The parametric resonance of density perturbation is discussed in Sec.III. We show that the efficiency of the parametric amplification on the spectrum of the curvature perturbation depends on the energy scale of the oscillating inflation and the growth factor. Section IV is devoted to the analysis of the growth factor. Neglecting the metric perturbation, we analytically find that the instability band of the parametric resonance is very broad and the growth factor may become significantly large. We then proceed the numerical calculation of cosmological perturbation without any approximation. The numerical analysis show that the perturbation can experience the parametric amplification, however, the amplification is strongly suppressed when the wavelength of the fluctuation approaches the Hubble horizon scales. The results are briefly summarized and the conclusion in the single field case is described in the last section V. We also discuss the oscillating inflation in the multi-field system. It could be expected that the significant amplification of the density perturbation can appear on large scales and may lead to the production of the cosmological black holes.

II. OSCILLATING INFLATION MODEL

Let us consider the minimally coupled scalar field ϕ in the flat FRW universe. The homogeneous background equations become

$$3H^2 = \frac{1}{\tilde{M}_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \quad ; \qquad \tilde{M}_{pl} = \sqrt{\frac{3}{8\pi}}\, M_{pl}, \tag{1}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{2}$$

where H is the Hubble parameter and M_{pl} is the Planck mass. The evolution of the scalar field depends on the shape of the potential $V(\phi)$. We shall consider the model of *oscillating inflation* proposed by Damour and Mukhanov [9], in which the potential is given by

$$V(\phi) = \frac{A}{q}\left[\left(\frac{\phi^2}{\phi_c^2} + 1\right)^{q/2} - 1\right]. \tag{3}$$

Note that the potential (3) becomes logarithmic, $V = (A/2)\log(\phi^2/\phi_c^2 + 1)$ in the limit $q \to 0$.

When the effective mass of the inflaton is larger than the Hubble parameter, i.e, $|V_{,\phi\phi}| \gg H^2$, the inflaton field shows oscillatory behavior. For the large amplitude $\phi \gg \phi_c$, taking the time average during a period of oscillation yields the following virial theorem:

$$\langle \dot{\phi}^2 \rangle \simeq q\, \langle V(\phi) \rangle. \tag{4}$$

Using this relation, the inflaton field can be regarded as the perfect fluid matter with the equation of state $P = (\gamma_{osc} - 1)\rho$, where γ_{osc} is the effective adiabatic index given by

$$\gamma_{osc} = \frac{2q}{q+2}. \tag{5}$$

Thus the time dependences of the scale factor, the amplitude of the inflaton field and inflaton potential are obtained [9] [11]:

$$\begin{aligned} a(t) &\propto t^{(q+2)/3q}, \\ \bar{\phi}(t) &\propto a^{-6/(q+2)} \propto t^{-2/q}, \\ V(\bar{\phi}) &\propto a^{-6q/(q+2)} \propto t^{-2}, \end{aligned} \tag{6}$$

where $\bar{\phi}$ is the amplitude of the scalar field evaluated at the time that $\dot{\phi} = 0$ holds.

Eq.(6) implies that if the potential index q is smaller than unity, the universe can experience the accelerated expansion even in the oscillating phase. When the scalar field approaches to the critical value $\bar{\phi} \simeq \phi_c$, the oscillating inflation ends. Hereafter, we shall restrict our attention on the inflationary phase with the potential index $q < 1$.

III. COSMOLOGICAL PERTURBATION

We now investigate the cosmological perturbation in the oscillating inflation. We shall ignore the interaction with the other scalar fields which leads to the inflaton decay in the reheating process [1–3]. In Ref. [5,6], the useful and well-behaved perturbed quantity during the oscillating stage is found :

$$Q = \delta\phi - \frac{\dot{\phi}}{H}\mathcal{R}, \tag{7}$$

where $\delta\phi$ and $\mathcal{R}$ are the perturbation of the scalar field and the spatial curvature, respectively [12]. The quantity Q is the gauge-invariant variable introduced by Mukhanov [13,14] and the evolution equation is simply given by

$$\ddot{Q} + 3H\dot{Q} + \left[\left(\frac{k}{a}\right)^2 + V_{,\phi\phi} + 6\tilde{M}_{pl}^{-2}\left(\frac{V(\phi)}{H}\right)^{\cdot}\right]Q = 0. \tag{8}$$

If we neglect the third term in the bracket, the above equation is just reduced to the equation for perturbation $\delta\phi$ ignoring the metric perturbation. We thus understand that the effect of gravitational interaction to the scalar field perturbation is encoded in the third term in the bracket.

In equation (8), there exists the exact solution in the long-wavelength limit [8]. For $k \to 0$, we have

$$Q = c_1\frac{\dot{\phi}}{H} + c_2\frac{\dot{\phi}}{H}\int\frac{dt}{a^3}\frac{H^2}{\dot{\phi}^2}, \tag{9}$$

where c_1 and c_2 are the integration constants. The solution proportional to the coefficient c_1 is named as the growing mode and the one proportional to c_2 as the decaying mode. † For the growing mode, the curvature perturbation on comoving slice $\mathcal{R}_c$ defined by

$$\mathcal{R}_c = \frac{H}{\dot{\phi}}Q \tag{10}$$

remains constant in time. The curvature perturbation is also referred to as the Bardeen parameter, which is simply related to the CMB temperature fluctuation $\Delta T/T$ observed by the Cosmic Background Explorer satellite [19]. In this paper, matching the amplitude of quantum fluctuation Q produced during the oscillating inflation inside the horizon with that of the long wavelength growing mode, we will evaluate the power spectrum of the curvature perturbation $\mathcal{P}_{\mathcal{R}}(k)$.

We focus on the evolution of the quantity Q on sub-horizon scales. In equation (8), the time dependence of the terms in the bracket is estimated by using the results (6):

$$V_{,\phi\phi}(\bar{\phi}) \qquad \propto\ a^{-6\left(\frac{q-2}{q+2}\right)}, \tag{11}$$

$$\left(\frac{V(\phi)}{H}\right)^{\cdot}\Bigg|_{\phi=\bar{\phi}} \simeq \frac{V_{,\phi}\dot{\phi}}{H}\Bigg|_{\phi=\bar{\phi}} \propto\ a^{-6\left(\frac{q-1}{q+2}\right)}, \tag{12}$$

which are valid for $\bar{\phi} \gg \phi_c$. During the oscillating inflation, the second term in the bracket becomes dominant compared to the term (12). The dominant contribution of the oscillating term (11) differs from the situation considered in Ref. [5] [6]. We then introduce the following variables:

$$Q = \bar{\phi}(t)\tilde{Q}(\eta),$$
$$d\eta = g(t)dt \quad ; \ g(t) = \frac{\sqrt{A}}{\bar{\phi}}\left(\frac{\bar{\phi}}{\phi_c}\right)^{q/2} \ \propto\ a^{-3\left(\frac{q-2}{q+2}\right)}.$$

Ignoring the term (12), the evolution equation (8) is rewritten as the normal form :

† It seems that the singular behavior appears in the decaying mode around the zero points of the time derivative of the scalar field. However, the zero points of the denominator in the integral can be canceled out by the zero points of $\dot{\phi}$ in the numerator. Therefore the solution is regular and the amplitude of the solution decreases in time. For more rigorous proof of the regularity and the explicit calculation of the long-wavelength perturbation, see [5] and [6].

$$\tilde{Q}_{,\eta\eta} + \left[\left(\frac{k}{ga} \right)^2 + \tilde{V}_{,\tilde{\phi}\tilde{\phi}}(\tilde{\phi}) \right] \tilde{Q} = 0, \tag{13}$$

where $\tilde{\phi}$ is defined by $\phi(t) = \bar{\phi}\ \tilde{\phi}(\eta)$ and satisfies the equation of motion

$$\tilde{\phi}_{,\eta\eta} + \tilde{V}_{,\tilde{\phi}}(\tilde{\phi}) = 0, \tag{14}$$

with the initial condition $\tilde{\phi}(\eta_i) = 1$. The quantities $\tilde{V}_{,\tilde{\phi}\tilde{\phi}}$ and $\tilde{V}_{,\tilde{\phi}}$ are obtained by differentiating the effective potential $\tilde{V}$ with respect to $\tilde{\phi}$:

$$\tilde{V}(\tilde{\phi}) = \frac{1}{q} \left[\left(\tilde{\phi}^2 + \frac{\phi_c^2}{\bar{\phi}^2} \right)^{q/2} - \left(\frac{\phi_c}{\bar{\phi}} \right)^q \right]. \tag{15}$$

Since we have $\tilde{V} \simeq \tilde{\phi}^q/q$, $\tilde{\phi}$ oscillates rapidly during the oscillating inflation. Treating the time dependence of the term $\tilde{V}$ adiabatically, the period of oscillation $\tilde{\phi}$ is estimated as

$$T = 4 \int_0^1 \frac{d\tilde{\phi}}{\sqrt{2(q^{-1} - \tilde{V})}} \simeq \sqrt{\frac{8\pi}{q}} \frac{\Gamma(\frac{1}{q})}{\Gamma(\frac{1}{q} + \frac{1}{2})}. \tag{16}$$

For the range $0.01 \le q \le 1$, we have $T \simeq 5$.

According to the Floquet theorem, presence of the oscillating term in (13) implies that there exists the following solution for the equation (13). Since $\tilde{V}_{,\tilde{\phi}\tilde{\phi}}$ is given by $\eta = T/2$, we have [21]

$$\tilde{Q}(\tau) = e^{\mu\tau}\ P(\tau)\ \ ;\ P(\tau + \pi) = P(\tau), \tag{17}$$

where we define the new time parameter $\tau \equiv (2\pi/T)\eta$. The periodic function $P(\tau)$ is bounded and has a finite amplitude. The characteristic exponent μ takes either a imaginary or a real number, which depends on the shape of $\tilde{V}_{,\tilde{\phi}\tilde{\phi}}$ and the parameter k/ga. The parametric resonance means that the characteristic exponent has a real number.

We can see that the parametric resonance can affect the curvature perturbation $\mathcal{R}_c$ produced during the oscillating inflation as follows. As usual, in the slow-rolling inflation, the amplitude of quantum fluctuation is given by $Q \simeq |\delta\phi| \sim H/2\pi$, which can be deduced from the instability of the massless scalar field in de Sitter space [15]. On the other hand, the amplification of the quantum fluctuations can appear due to the parametric resonance in the oscillating inflation. We have

$$\left| \frac{\tilde{Q}_*}{\tilde{Q}_0} \right| \simeq e^{\mu\Delta\tau}, \tag{18}$$

where subscripts $(_0)$ and $(_*)$ denotes the quantities evaluated at the beginning of oscillating inflation and at the time when the wavelength of the quantum fluctuation exceeds the Hubble horizon scale ($k = a_* H_*$), respectively. Here, $\Delta\tau$ denotes the time interval during which the quantum fluctuation is amplified by the parametric resonance. For the modes inside the horizon $k/aH \to \infty$, the mode function of the equation (8) initially becomes $a_0 Q_0 \to \exp\left(i \int dt/a\right)/\sqrt{2k}$, corresponding to the vacuum state in the Minkowski spacetime [16]. Matching the amplitude of the short wavelength fluctuation with that of the long wavelength solution (9), the curvature perturbation (10) is roughly estimated :

$$|\mathcal{R}_c| \simeq \left(\frac{H}{\dot{\phi}} \right)_* \frac{H_*}{\sqrt{2k^3}} \left(\frac{\bar{\phi}_*}{\bar{\phi}_0} \right)^{(4-q)/(2+q)} \left| \frac{\tilde{Q}_*}{\tilde{Q}_0} \right|, \tag{19}$$

which is evaluated at the horizon-crossing time. Therefore, the power spectrum of the curvature perturbation $\mathcal{P}_\mathcal{R}(k)$ evaluated by taking the ensemble average $\langle \cdots \rangle_{ens}$ becomes

$$\mathcal{P}_\mathcal{R}^{1/2}(k) \equiv \sqrt{\frac{k^3}{2\pi^2}} \langle |\mathcal{R}_c|^2 \rangle_{ens}^{1/2} \simeq \frac{\gamma_{osc}^{-1}}{2\pi} \left(\frac{\bar{\phi}_*}{\bar{\phi}_0} \right)^{(4-q)/(2+q)} \left(\frac{V_*^{1/4}}{\tilde{M}_{pl}} \right)^2\ e^{\mu\Delta\tau}. \tag{20}$$

In the derivation of the expression (20), we have naively replaced the ensemble average $\langle \dot{\phi}^2 \rangle_{ens}$ with the time average $\langle \dot{\phi}^2 \rangle$ and used the virial theorem (4). Eq.(20) shows that the appearance of exponential factor $e^{\mu\Delta\tau}$ may give the important contribution to the spectrum of curvature perturbation, although the amplitude of $\mathcal{P}_\mathcal{R}$ is always suppressed by the energy scale of the oscillating inflation $V_*^{1/4} \equiv V^{1/4}(\bar{\phi}_*)$ [16] [19].

IV. PARAMETRIC AMPLIFICATION DURING OSCILLATING INFLATION

In the previous section, we have seen that the effect of parametric resonance can lead to the amplification of the curvature perturbation, which may gives $\mathcal{P}_{\mathcal{R}} \sim 1$. In this section, to explore the efficiency of the parametric amplification, we analytically and numerically investigate the growth factor $e^{\mu\Delta\tau}$.

A. Analytic estimation

Let us evaluate the characteristic exponent μ from the reduced equation (13). Using the solution (17) obtained from the Floquet theorem, we have the following formula [21]:

$$\mu = \frac{1}{\pi}\log\left[\sqrt{F^2} + \sqrt{F^2-1}\right] \quad ; \quad F = 1 + 2\tilde{Q}_1'(\tau = \frac{\pi}{2})\tilde{Q}_2(\tau = \frac{\pi}{2}), \tag{21}$$

where the prime denotes the differentiation with respect to the time τ. $\tilde{Q}_1$ and $\tilde{Q}_2$ are the solutions satisfying the initial conditions $\tilde{Q}_1 = 1$, $\tilde{Q}_1' = 0$ and $\tilde{Q}_2 = 0$, $\tilde{Q}_2' = 1$ at $\tau = 0$, respectively. The parametric resonance appears in the case $F > 1$.

Although it is difficult to obtain the solutions $\tilde{Q}_1$ and $\tilde{Q}_2$ for our complicated potential $\tilde{V}_{,\tilde{\phi}\tilde{\phi}}$, we can get the approximate expression of the characteristic exponent. The effective mass of the oscillating inflaton field becomes negative at the shallow wings of the potential and positive at the minimum. Thus we replace the periodic function $\tilde{V}_{,\tilde{\phi}\tilde{\phi}}$ with the *step-wise* function given by

$$\tilde{V}_{,\tilde{\phi}\tilde{\phi}} \longrightarrow \begin{cases} \tilde{V}_{,\tilde{\phi}\tilde{\phi}}(\tilde{\phi} = 0), & (|\tau| < \tau_1) \\ \tilde{V}_{,\tilde{\phi}\tilde{\phi}}(\tilde{\phi} = 1), & (\tau_1 < |\tau| < \frac{\pi}{2}) \end{cases}, \tag{22}$$

where we choose

$$\tau_1 = \frac{2\pi}{T}\int_0^{\tilde{\phi}_1} \frac{d\tilde{\phi}}{\sqrt{2(q^{-1} - \tilde{V})}} \quad ; \quad \tilde{\phi}_1 = \sqrt{\frac{3}{1-q}}\frac{\phi_c}{\bar{\phi}}.$$

At $\tilde{\phi} = \tilde{\phi}_1$, we have $\tilde{V}_{,\tilde{\phi}\tilde{\phi}\tilde{\phi}} = 0$. The approximation (22) enables us to obtain the analytic solutions $\tilde{Q}_1$ and $\tilde{Q}_2$. Then the quantity F is evaluated as follows:

$$F = \frac{1}{2}\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)\sin(2\alpha\tau_1)\sin(2\beta(\tau_1 - \frac{\pi}{2})) + \cos(2\alpha\tau_1)\cos(2\beta(\tau_1 - \frac{\pi}{2})), \tag{23}$$

where

$$\alpha = \frac{T}{2\pi}\left[\left(\frac{k}{ga}\right)^2 + \tilde{V}_{,\tilde{\phi}\tilde{\phi}}(\tilde{\phi} = 0)\right]^{1/2}, \qquad \beta = \frac{T}{2\pi}\left[\left(\frac{k}{ga}\right)^2 + \tilde{V}_{,\tilde{\phi}\tilde{\phi}}(\tilde{\phi} = 1)\right]^{1/2}.$$

Since $\tilde{V}_{,\tilde{\phi}\tilde{\phi}}(\tilde{\phi} = 1) < 0$ during the oscillating inflation, the variable β becomes imaginary for the long-wave modes $(k/a)^2 \lesssim |V_{,\phi\phi}(\bar{\phi})|$ and we obtain $F > 1$. This can be deduced from the negative coupling instability [9]. Important point is that we could also have $F > 1$ for the short-wave mode $(k/a)^2 \gtrsim |V_{,\phi\phi}(\bar{\phi})|$. In the latter case, the band structure of the characteristic exponent μ appears.

In Fig.1, we plot the characteristic exponent μ as a function of k/ga evaluated from (21) and (23). The parameters are chosen as $\phi_c = 10^{-6}\ \tilde{M}_{pl}$ and $q = 0.1$. As for the quantity $\bar{\phi}$, we treat it as constant and set $\bar{\phi} = q\ \tilde{M}_{pl}/\sqrt{6}$, corresponding to the initial value of the oscillating inflation [10]. The circles show the characteristic exponent obtained from the substitution of the numerical solutions of equations (13) and (14) into the formula (21). We see that the approximation predicts the numerical result reasonably well. The crucial observation is that the instability band is very broad and the characteristic exponent can be of the order of unity.

To evaluate the growth factor $e^{\mu\Delta}$, we further need the period $\Delta\tau$. It is given by

$$\Delta\tau = \frac{2\pi}{T}\int dt g(t) = \sqrt{\frac{\pi}{72}}\ q(q+2)\frac{\Gamma(\frac{1}{q} + \frac{1}{2})}{\Gamma(\frac{1}{q})}\left(\frac{\tilde{M}_{pl}}{\bar{\phi}_*} - \frac{\tilde{M}_{pl}}{\bar{\phi}_0}\right), \tag{24}$$

where the integral is evaluated from the time at which the oscillating inflation starts to the horizon-crossing time. Eq.(24) is valid for the case $q > 0$. Rewriting the above equation with the relation $k = a_* H_*$, the wave number dependence of the growth factor $e^{\mu\Delta\tau}$ shows that the characteristic peak appears on the spectrum of curvature perturbation (20). Because of the rapid oscillation of the inflaton field, the period $\Delta\tau$ effectively becomes very large. For $q = 0.1$, the typical values $\bar{\phi}_* = 10^{-4}$, 10^{-5} and 10^{-6} $\tilde{M}_{pl}$ give $\Delta\tau = 1367$, 13696 and 136991, respectively. Therefore, even if we get the rather small value of the characteristic exponent $\mu = 0.01$, for example, the contribution of the growth factor to the curvature perturbation becomes significantly large. For $\bar{\phi}_* = 10^{-5}\tilde{M}_{pl}$ and $q = 0.1$, we have $e^{\mu\Delta\tau} = 10^{182}$!

B. Numerical result

The analytic estimation in the previous subsection rather overestimates the growth factor. For more rigorous evaluation, we must take into account the time dependence of $\bar{\phi}$ and k/ga. Furthermore, we should remember that the previous result comes from the analysis of the reduced equation (13). This also leads to the overestimation of the growth factor. However, there still exists a possibility of the parametric amplification of the curvature perturbation spectrum. To clarify the influence of parametric resonance, we numerically solve the equations (1), (2) and (8).

Fig.2 shows the growth factor $e^{\mu\Delta\tau_e}$ for the index $q = 0.1$, i.e, the ratio of the amplitude $\tilde{Q}_e$ evaluated at the end of oscillating inflation to the amplitude $\tilde{Q}_0$ at the beginning of inflation by varying the parameter ϕ_c. For each value ϕ_c, we start to calculate the background evolution by setting the value $\phi_0 = q\tilde{M}_{pl}/\sqrt{6}$ corresponding to the final value of the slow-rolling inflation. As for the fluctuations, the initial conditions $\tilde{Q}'_0 = 0$ are chosen. Although we have checked the evolution for the various initial conditions $\tilde{Q}_0 \propto \cos(k\int dt/a)$, corresponding to the vacuum state in the short wavelength limit, the maximal amplitude can be obtained from the initial condition $\tilde{Q}'_0 = 0$. Fig.2 shows that the amplification for the fluctuations occurs by the effect of parametric resonance. It is obvious that the maximal amplitude depends on the parameter ϕ_c. For the smaller value of ϕ_c, the amplitude of the fluctuation becomes larger, which comes from the fact that the period of inflation represented by the e-folding number $N_e = \log_e(a_e/a_0)$ depends on the critical value ϕ_c [9,10]. When the oscillating inflation takes place for a longer time, the effect of parametric resonance works out more efficiently and the fluctuations are significantly amplified. In Fig.2, the three different lines are depicted. The solid line is the fluctuation with the wavelength ten times smaller than the radius of the Hubble horizon at the end of oscillating inflation, i.e, $k = 10a_eH_e$. The dashed line has the wavelength $k = 2a_eH_e$ and the dotted line corresponds to the fluctuation whose wavelength just reaches at the Hubble radius after oscillating inflation. It is remarkable that the modes inside the Hubble horizon are enormously amplified, while the amplification of the long-wave mode comparable to the Hubble horizon is strongly suppressed. Though the dotted line apparently takes the growth factor $\sim 10^7$, it can be recognized that the parametric resonance becomes ineffective for the long-wave modes.

To see the efficiency of the parametric amplification explicitly, we plot in Fig.3 the time evolution of the fluctuations using the quantity Q from the beginning to the end of oscillating inflation. We set the parameters $q = 0.1$ and $\phi_c = 10^{-5}\tilde{M}_{pl}$. The horizontal axis represents the cosmic time normalized by the factor $\sqrt{A}/\tilde{M}_{pl}$. Each figure shows the time evolution with the different wavelength under the same initial conditions $Q_0 = 1.0$ and $Q'_0 = 0$: (a) $k = 10a_eH_e$;(b) $k = 2a_eH_e$; (c) $k = a_eH_e$. These figures clearly reveal that the growth of the perturbation is much sensitive to the wavelength of the fluctuations. The parametric amplification works efficiently inside the horizon, however, the amplitude Q becomes constant for the wavelength near the Hubble horizon.

We can understand these behavior as follows. As was described in Sec.III, there exists the exact solution for the variable Q in the limit $k \to 0$. Ignoring the decaying mode proportional to the coefficient c_2 in (9), the amplitude of the long wavelength solution becomes nearly constant, which can be deduced from (6). This behavior can also be obtained in the case of $k \neq 0$, when the term $(k/a)^2$ in equation (8) is negligible compared to the terms $V_{\phi\phi}$ and $\tilde{M}^{-2}(V/H)^{\cdot}$, which gives the condition $k/a \ll H$ [6]. In the previous subsection, we analyzed the growth factor $e^{\mu\Delta\tau}$ for the short-wave mode by dropping the term $\tilde{M}^{-2}(V/H)^{\cdot}$. Fig.3 indicates that the term induced by the gravitational perturbation plays a crucial role even for the modes comparable to the Hubble horizon. This is fully consistent with the result of the paper [5] and [6]. Therefore, the spectrum of the curvature perturbation produced during oscillating inflation on super-horizon scales has the rather small amplitude compared with the analytic prediction.

V. CONCLUDING REMARKS

We have analyzed the cosmological perturbation in the oscillating inflation and explored a possibility of parametric amplification for the curvature perturbation. Neglecting the metric perturbation, the analytic estimation shows

that the curvature perturbation could be amplified by the broad band parametric resonance. We then numerically examined this issue by solving the evolution equation for perturbation without any approximation. The enormous amplification of the perturbation can appear, however, we found that the presence of the the metric perturbation strongly suppresses the parametric amplification on large scales comparable to the Hubble horizon.

Now we discuss the cosmological implication of the parametric resonance to the density perturbation. According to Ref. [18,19], the physical length of the fluctuation produced during the oscillating inflation is

$$\lambda_{phy} \lesssim 10^{1+0.43\alpha_*} \;\; \mathrm{cm}, \tag{25}$$

where λ_{phy} denotes the present wavelength. The quantity α_* is given by

$$\alpha_* = N_* + \log_e \left(\frac{10^{16}\ \mathrm{GeV}}{V_*^{1/4}} \right). \tag{26}$$

N_* denotes the e-folding number of the oscillating inflation evaluated at the Hubble crossing time. On the other hand, if the amplitude $\mathcal{R}_c$ produced during the inflation becomes of the order of unity, the fluctuations can experience the gravitational collapse. Assuming that the primordial black holes are formed at the horizon re-entry time during the radiation epoch, we can evaluate the typical mass of a black hole using the equation (25). We obtain

$$M_{BH} \lesssim 10^{1+0.86\alpha_*} \;\; \mathrm{g}, \tag{27}$$

which is evaluated at the formation epoch.

As is shown by Liddle and Mazumdar [10], a period of oscillating inflation is short. This can be checked in our numerical calculation. For the critical value $\phi_c = 10^{-6}\tilde{M}_{pl}$, we obtain the small value of the e-folding number $N_e = 3.5$. Hence, the expressions (25) and (27) show that the energy scale of the oscillating inflation should be rather low to have the cosmologically interesting scale for the back hole mass and the physical size of the fluctuations. Asaka, Kawasaki and Yamaguchi [20] investigated the inflation model to resolve the cosmological moduli problem, in which the oscillating inflation occurs at the energy scale $V_e^{1/4} \simeq 10$GeV, instead of the thermal inflation. In this energy scale, if we have the e-folding number $N_e \simeq 3$, it is possible to get $M_{BH} \sim 1M_\odot$, as a candidate for explaining the recent observation of the massive compact halo objects, although the physical length is still small, $\lambda_{phy} \sim 1\ pc$.

However, the amplitude of the vacuum fluctuation becomes extremely small for the inflation at low energy scale. We saw in Sec.III that the spectrum of curvature perturbation $\mathcal{P}_\mathcal{R}$ given by (20) contains the suppression factor $(V_*^{1/4}/M_{pl})^2$, which takes $\sim 10^{-36}$ in the above case. Furthermore, our results show that the gravitational interaction plays a crucial role for the parametric amplification of the curvature perturbation. To obtain the relevant growth factor $e^{\mu\Delta\tau}$ to compensate the suppression factor, a long period of oscillating inflation is necessary, which requires the very fine-tuning of the model parameter ϕ_c. Therefore, in the single field case, we conclude that the influence of the parametric resonance on the cosmological perturbation would not be imprinted on the universe observed as long as the adiabatic mode of the perturbation is dominant.

The conclusion might be changed if we consider the multi-field system. Consider the hybrid-type inflation which is a more realistic model motivated by the particle physics [19]. In this scenario, we can imagine that the oscillating inflation occurs subsequent to the slow-rolling inflation and it is followed by a secondary inflation driven by another scalar field. If we choose the appropriate duration of the secondary inflation, the oscillating inflationary phase could have the relevant length scales λ_{phy} which can affect the large scale structure. In addition, the hybrid inflation would make a modification of the spectrum $\mathcal{P}_\mathcal{R}$. In equation (20), we should replace the term $(V_*^{1/4}/\tilde{M}_{pl})^2$ with $V_*^{1/4}/\tilde{M}_{pl}$ [17]. This relaxes the fine-tuning of the model parameter ϕ_c to give the cosmologically interesting scale of the black hole mass M_{BH}. The effect of oscillating inflation might be imprinted on the primordial density perturbation in such inflationary scenario. To investigate the spectrum $\mathcal{P}_\mathcal{R}$ of the hybrid inflation, we must study the cosmological perturbations with multi-field system, in which the contribution of iso-curvature perturbation cannot be neglected [7]. The non-linear interaction of the oscillating inflaton with the secondary inflation-driven field would lead to the significant amplification of the iso-curvature perturbation by the parametric resonance. Although the influence of iso-curvature mode on the gauge-invariant perturbation $\mathcal{R}_c$ has not been understood completely, the result in this paper would shed light on the evaluation of the primordial power spectra. We will report the analysis of the multi-field system in a separate publication.

ACKNOWLEDGMENTS

The author would like to thank M.Sakagami for providing him the numerical code and useful comments, J.Soda for careful reading of the manuscript, S.Kawai and K.Koyama for valuable discussions and comments.

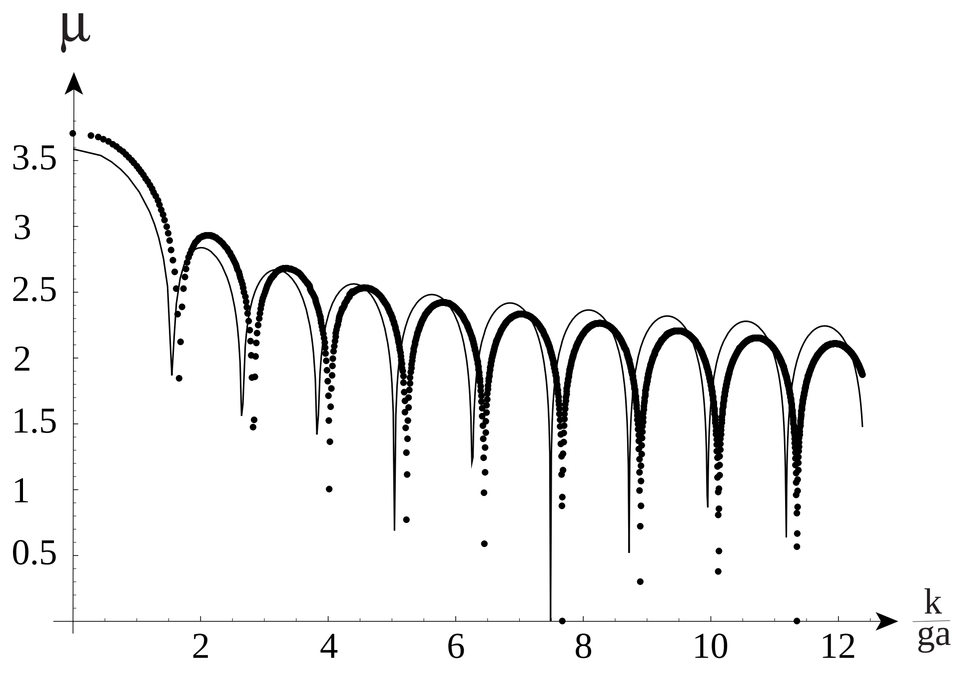
[1] L.Kofman, A.Linde and A.A.Starobinsky, Phys.Rev.Lett.**73**, 3195 (1994).
[2] L.Kofman, A.Linde and A.A.Starobinsky, Phys.Rev.D.**56**, 3258 (1997).
[3] P.B.Greene, L.Kofman, A.Linde and A.A.Starobinsky, Phys.Rev.D.**56**, 6175 (1997).
[4] E.W.Kolb, A.Linde and A.Riotto, Phys.Rev.Lett.**77**, 4290 (1996).
[5] H.Kodama and T.Hamazaki, Prog.Theor.Phys.**96**, 949 (1996).
[6] Y.Nambu and A.Taruya, Prog.Theor.Phys.**97**, 83 (1997).
[7] A.Taruya and Y.Nambu, Phys.Lett.B**428**, 37 (1998).
[8] Y.Nambu and A.Taruya, Class.Quantum.Grav.**15**, 2761 (1998).
[9] T.Damour and V.F.Mukhanov, Phys.Rev.Lett.**80**, 3440 (1998).
[10] A.R.Liddle and A.Mazumdar, Phys.Rev.D**58**, 083508 (1998).
[11] M.S.Turner, Phys.Rev.D**28**, 1243 (1983).
[12] H.Kodama and M.Sasaki, Prog.Theor.Phys.Suppl.**78**, 1 (1984).
[13] V.F.Mukhanov, Sov.Phys.JETP**67**, 1297 (1998).
[14] V.F.Mukhanov, H.A.Feldman and R.H.Brandenberger, Phys.Rep.**215**, 203 (1992).
[15] A.D.Linde, *Inflation and Quantum Cosmology* (Academic Press, Inc. 1990).
[16] E.D.Stewart and D.H.Lyth, Phys.Lett.B**302**, 171 (1993).
[17] L.Randall, M.Soljacic and A.H.Guth, Nulc.Phys.B**472**, 377 (1996).
[18] A.R.Liddle and D.H.Lyth, Phys.Rep.**231**, 1 (1993).
[19] D.H.Lyth adn A.Riotto, hep-ph/ 9807278.
[20] T.Asaka, M.Kawasaki and M.Yamaguchi, hep-ph/ 9810334.
[21] M.Abramowitz and I.Stegun, *Handbook of Mathematical Functions* (Dover,New York, 1965), p.721.

FIGURE CAPTION

Fig1 Characterisctic exponent μ for the parameters $q = 0.1$ and $\phi_c = 10^{-6}\tilde{M}_{pl}$. We plot the exponent μ as a function of the Fourier mode k/ga. The solid line shows the approximation using the result (23). The circles are the characteristic exponent evaluated by substituting the numerical solutions of eqs.(13) and (14) into the formula (21). In both cases, the variable $\bar{\phi}$ in the effective potential $\tilde{V}$ is specified as $\bar{\phi} = q\tilde{M}_{pl}/\sqrt{6}$.

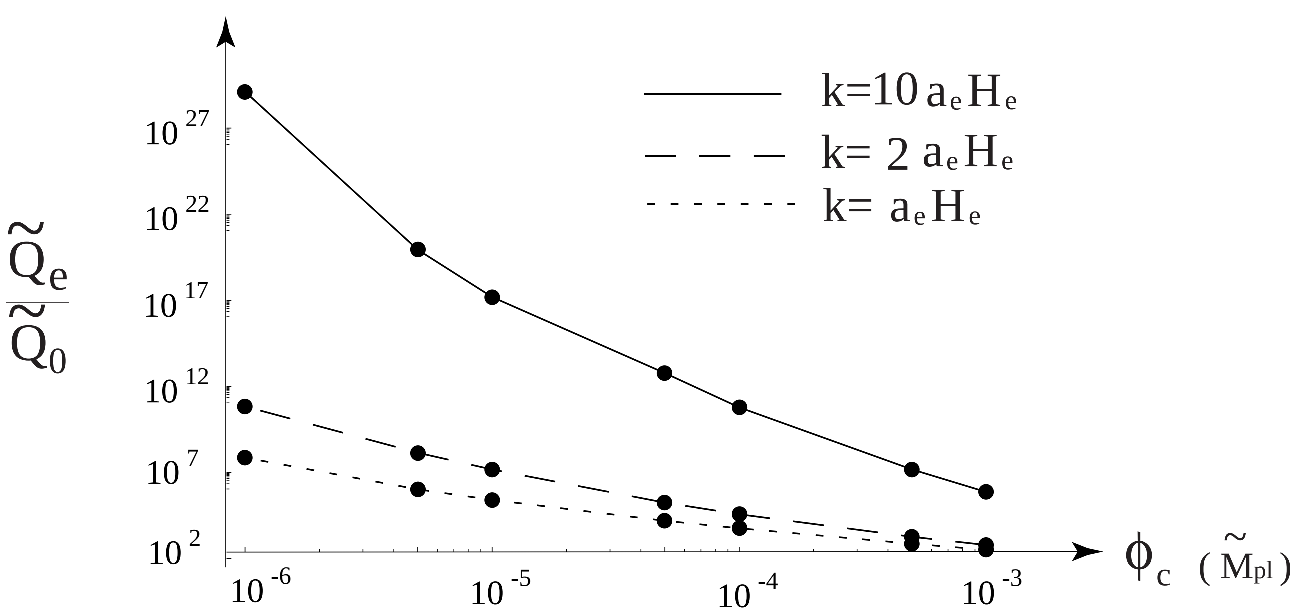
Fig2 The growth factor $e^{\mu\Delta\tau_e}$, the ratio of the amplitude $\tilde{Q}_e$ evaluated at the end of the oscillating inflation to the one $\tilde{Q}_0$ at the beginning of inflation. We plot the ratio of the amplitude with the same initial condition $\tilde{Q}'_0 = 0$ by varying the model parameter ϕ_c. We set the potential index $q = 0.1$. The solid line corresponds to the fluctuation with the mode $k = 10a_eH_e$, i.e, the wavelength is ten times smaller than the Hubble horizon size at the end of oscillating inflation. The dashed line represents the amplitude for the modes $k = 2a_eH_e$. The dotted line is the ratio for the fluctuation whose wavelength reaches at the Hubble horizon size just after the oscillating inflation ends.

Fig3 Evolution of Q in the case of the parameters $q = 0.1$ and $\phi_c = 10^{-5}\tilde{M}_{pl}$. For each figure, we start to calculate the background eqs.(1) and (2) from the value $\phi = q/\sqrt{6}$ after slow-rolling regime. The initial conditions for the fluctuation Q are set by $Q = 1.0,\ Q' = 0$ and eq.(8) is solved numerically. The horizontal axis denotes the cosmic time normalized by $\sqrt{A}/\tilde{M}_{pl}$: (a) The fluctuation Q for the mode $k = 10a_eH_e$; (b) The fluctuation Q for the mode $k = 2a_eH_e$; (c) The gauge-invariant quantity Q whose wavelength reaches at the Hubble horizon size just after the oscillating inflation ends, i.e, $k = a_eH_e$.



μ
3.5
3
2.5
2
1.5
1
0.5
2
4
6
8
10
12
k/ga

Fig.1



k=10 aₑHₑ
k= 2 aₑHₑ
k= aₑHₑ
Q̃ₑ/Q̃₀
10^27
10^22
10^17
10^12
10^7
10^2
10^-6
10^-5
10^-4
10^-3
φc (M̃pl)

Fig.2

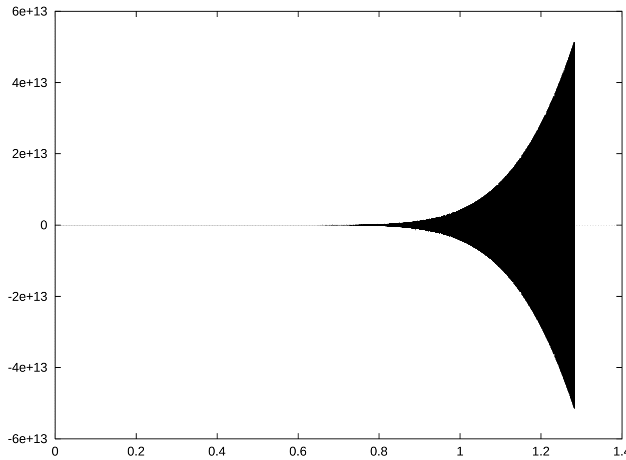


Fig.3a

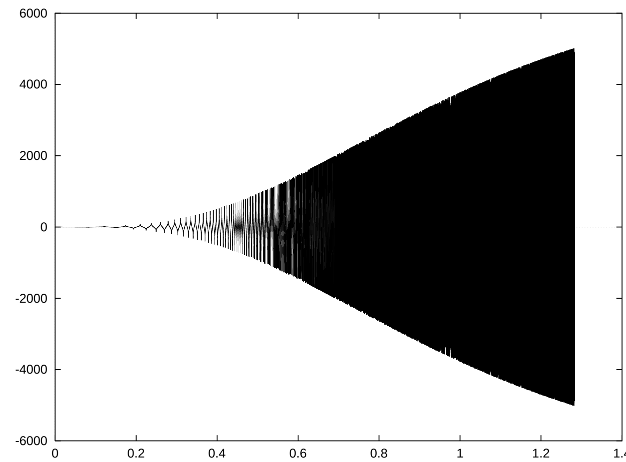


Fig.3b

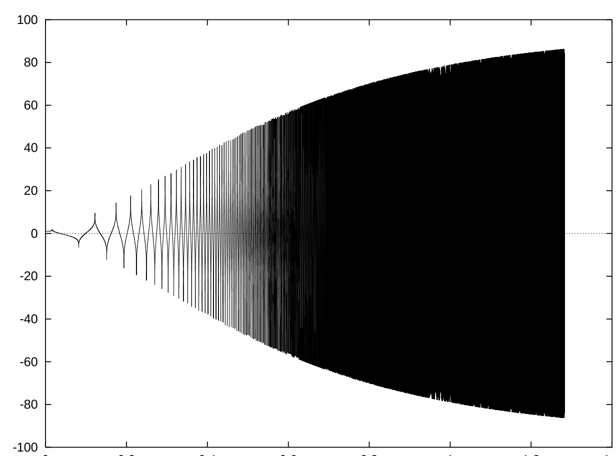


Fig.3c